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The Almost (E,q) (N,P_n) Summability of Fourier Series

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Abstract: **The degree of approximation of function $f \in$ almost lips α by the $(E,q)(N,P_n)$ means of Fourier series is determined.**

Keywords: **degree of Approximation, class L^aip α , (E,q)summability, (N,Pn) summability Product summability, Lebesgue integral.**

I. INTRODUCTION

Let $\sum an$ be a given infinite series with the sequence of partial sums$\{S_n\}$. Let $\{P_n\}$ be a sequence of positive real numbers such that

$$P_n = \sum_{v=0}^{n} P_v \to \infty \, as \, n \to \infty \, (P_{-i} = P_i = 0, \; i \geq 0) \qquad \ldots.\ (1.1)$$

The sequence to sequence transformation

$$t_n = \frac{1}{Pn} \sum_{v=0}^{n} Pn - v \, Sv \qquad \ldots..\ (1.2)$$

Defines the sequence $\{t_n\}$ of the (N,P_n) means of the sequence $\{S_n\}$ generated by the sequence of coefficient$\{P_n\}$. If

$$t_n \to s \; as \; n \to \infty \qquad \ldots\ldots\ (1.3)$$

then the series $\sum a_n$ is said to be (N,P_n) summable to s.

The condition for regularity of Norlund Summ ability (N,P_n) are easily seen to be

(i) $\frac{p_n}{P_n} \to 0$ as $n \to \infty$

(ii) $\sum_{k=0}^{n} P_K = o(P_n)$ as $n \to \infty$.

The sequence to sequence transformation

$$T_n = \frac{1}{(1+q)^n} \sum_{v=0}^{n} \binom{n}{v} q^{n-v} \, \mathrm{Sv} \qquad \ldots\ldots.\ (1.4)$$

Defines the sequence $\{T_n\}$ of the (E,q) mean of the sequence $\{S_n\}$ If

$$T_n \to s \; as \; n \to \infty \qquad \ldots\ldots.\ (1.5)$$

then the series $\sum a_n$ is said to be (E,q) summable to s.

Clearly (E,q) method is regular.

The (E,q) transform of the (N,P_n) transform of $\{S_n\}$ is defined by

$$\tau_n = \frac{1}{(1+q)^n} \sum_{k=0}^{n} \binom{n}{k} q^{n-k} \quad T_K$$

$$= \frac{1}{(1+q)^n} \sum_{k=0}^{n} \binom{n}{k} q^{n-k} \left\{ \frac{1}{Pk} \sum_{v=0}^{k} P_{k-v} Sv \right\} \quad \ldots\ldots (1.6)$$

If $\tau_n \to s$ as $n \to \infty$ then $\sum a_n$ is said to be $(E, q)(N, P_n)$ summable to s.

Let $f(t)$ be a periodic function with period 2π L- integrable over$(-\pi, \pi)$. The Fourier series

Associated with f at any point x is defined by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \ldots\ldots (1.7)$$

And the conjugate series of the Fourier series is

$$\sum_{n=1}^{\infty} (a_n \cos nx - b_n \sin nx) \quad \ldots\ldots (1.8)$$

A function f € Lip α, if $|f(x+t) - f(x)| = o\,|t|^{\alpha}$ $0 < \alpha \leq 1$ ……. (1.9)

Let $0 < \alpha \leq 1$ and let $f: R \to R$ be almost Lipchitz of order α, f € L^aip α in the sense that there is a constant $M = M_f \geq 0$ and for each $x \in R$ there is a subset. $A_x \subset [0, \pi/2]$ of measure zero such that $t \in [0, \pi/2] \setminus A_x$ implies

$$|f(x+2t) - f(x-2t)| \leq Mt^{\alpha} \quad \ldots.. (1.10)$$

Every Lip α function is trivially Laipα, but the class Lip α greatly extends the class Lip α .For $0 < t \leq \pi/2$, since $sint \geq \frac{2t}{\pi}$ .So for each $x \in R$.

We have $|\Psi(t)\, cost| \leq Mt^{\alpha} \frac{\pi}{2t} = Mt^{\alpha-1} \frac{\pi}{2}$, $t \in [0, \pi/2] \setminus A_x$

Where $\Psi(t) = f(x+2t) - f(x-2t)$.

We use the following notation throughout this paper

$$\varphi(t) = f(x+t) - f(x-t) - 2f(x)$$

$$\Psi(t) = \tfrac{1}{2}\{f(x+t) - f(x-t)\} \text{ And}$$

$$K_n(t) = \frac{1}{2\pi(1+q)^n} \sum_{k=0}^{n} \binom{n}{k} q^{n-k} \left\{ \frac{1}{p_k} \sum_{v=0}^{k} P_{k-v} \frac{\sin(v+1/2)t}{\sin t/2} \right\}$$

II. MAIN THEOREM

Theorem 2.1– If f is a 2π periodic function of class L^aip α then t he degree of a approximation by the product (E,q) (N,P_n) summability mean on its Fourier series (1.7) is given by

$\|\tau_n - f\|_{\infty} = o\left(\frac{1}{(n+1)^{\alpha}}\right)$ $0 < \alpha < 1$ where τ_n on defined in (1.6).

1. **Lemma –** We require the following lemma to prove the theorem.

3.1 Lemma- $|Kn(t)| = o(n)$ $0 \leq t \leq \frac{1}{n+1}$

Proof – For $0 \leq t \leq \frac{1}{n+1}$ we have $\sin nt \leq n \sin t$ then

$$|Kn(t)| = \frac{1}{2\pi(1+q)^n} \left| \sum_{k=0}^{n} \binom{n}{k} q^{n-k} \left\{ \frac{1}{p_k} \sum_{v=0}^{k} P_{k-v} \frac{\sin(v+1/2)t}{\sin t/2} \right\} \right|$$

$$\leq \frac{1}{2\pi(1+q)^n} \left| \sum_{k=0}^{n} \binom{n}{k} q^{n-k} \left\{ \frac{1}{p_k} \sum_{v=0}^{k} P_{k-v} \frac{(2v+1)\sin t/2}{\sin t/2} \right\} \right|$$

$$\leq \frac{1}{2\pi(1+q)^n}\left|\sum_{k=0}^{n}\binom{n}{k}q^{n-k}(2k+1)\left\{\frac{1}{p_k}\sum_{v=0}^{k}P_{k-v}\right\}\right|$$

$$\leq \frac{(2n+1)}{2\pi(1+q)^n}\left|\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\right|$$

$$= o(n)$$

3.2 Lemma $|\,Kn(t)| = o\left(\frac{1}{t}\right)$ for $\frac{1}{n+1} \leq t \leq \pi$.

Proof- For $\frac{1}{n+1} \leq t \leq \pi$ we have $\sin(t/2) \geq t/\pi$, $\sin nt \leq 1$ then $|\,Kn(t)| = \frac{1}{2\pi(1+q)^n}$

$$\left|\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\left\{\frac{1}{p_k}\sum_{v=0}^{k}P_{k-v}\frac{\sin\left(v+\frac{1}{2}\right)t}{\sin\frac{t}{2}}\right\}\right|$$

$$\leq \frac{1}{2\pi(1+q)^n}\left|\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\left\{\frac{1}{p_k}\sum_{v=0}^{k}\frac{\pi}{t}P_{k-v}\right\}\right|$$

$$\leq \frac{1}{2t(1+q)^n}\left|\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\left\{\frac{1}{p_k}\sum_{v=0}^{k}P_{k-v}\right\}\right|$$

$$= \frac{1}{2t(1+q)^n}\left|\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\right|$$

$$= o\left(\frac{1}{t}\right).$$

Proof the theorem:

The n[th] partial sum Sn(x) of the fourier series (1.7) can be written as

$$S_n(x) - f(x) = \frac{1}{2\pi}\int_0^{\pi}\emptyset(t)\frac{\sin(n+\frac{1}{2})t}{\sin(t/2)}dt$$

The (N,Pn) transform of Sn(x) is given by

$$tn - f(x) = \frac{1}{2\pi P_n}\int_0^{\pi}\emptyset(t)\sum_{k=0}^{n}p_{n-k}\frac{\sin(n+\frac{1}{2})t}{\sin(t/2)}dt$$

The (E,q) (N, p_n) transform of Sn(x) is given by

$$\|\tau_n - f\| = \frac{1}{2\pi(1+q)^n}\int_0^{\pi}\emptyset(t)\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\left\{\frac{1}{p_k}\sum_{v=0}^{k}p_{k-v}\frac{\sin(v+\frac{1}{2})t}{\sin(t/2)}dt\right\}$$

$$= \int_0^{\pi}\emptyset(t)k_n(t)\,dt\}$$

$$= \left\{\int_0^{1/n+1} + \int_{1/n+1}^{\pi}\right\}\emptyset(t)k_n(t)dt$$

$$= I_1 + I_2 \qquad \text{……..(4.1)}$$

$$|I_1| = \frac{1}{2\pi(1+q)^n}\left|\int_0^{1/n+1}\emptyset(t)\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\left\{\frac{1}{p_k}\sum_{v=0}^{k}P_{k-v}\frac{\sin(v+1/2)t}{\sin t/2}\right\}dt\right|$$

$= o(n)\int_0^{1/n+1}|\emptyset(t)|\,dt$ By Lemma(3.1)

$$= o(n)\int_0^{1/n+1} M|t^{\propto}|\, dt$$

$$\leq o(n) \quad [\frac{t^{\propto+1}}{\propto+1}]_0^{1/n+1}$$

$$= o\left[\frac{1}{(n+1)^{\propto}}\right] \qquad \ldots\ldots\ldots(4.2)$$

$$|I_2| = \frac{1}{2\pi(1+q)^n}\left|\int_{1/n+1}^{\pi} \emptyset(t)\sum_{k=0}^{n}\binom{n}{k} q^{n-k}\left\{\frac{1}{p_k}\sum_{v=0}^{k} P_{k-v}\frac{\sin(v+1/2)t}{\sin t/2}\right\}dt\right|$$

$$\leq \int_{1/n+1}^{\pi}|\emptyset(t)|\,|K_n(t)|dt$$

$$= \int_{1/n+1}^{\pi}|\emptyset(t)|\, o(\tfrac{1}{t})dt \qquad \text{By Lemma} \quad (3.2)$$

$$\leq \int_{1/n+1}^{\pi} M|t^{\propto}|\, o(\tfrac{1}{t})dt$$

$$\leq \int_{1/n+1}^{\pi} t^{\propto-1}\, dt$$

$$= o\left(\frac{1}{(n+1)^{\propto}}\right) \qquad \ldots\ldots\ldots(4.3)$$

From (4.1) (4.2) and (4.3) we have

$$|\tau_n - f(t)| = o\left(\frac{1}{(n+1)^{\propto}}\right) \quad \text{for } 0 < \propto < 1.$$

Hence

$$\|\tau_n - f(x)\| = \sup_{-\pi < x < \pi}|\tau_n - f(x)| = o\left(\frac{1}{(n+1)^{\propto}}\right) \qquad 0 < \propto < 1.$$

This completes the proof of the theorem.

Corollary: If Pn = 1 $\forall n$ *and* q= 1 then theorem reduces to degree of approximation for (E,1) (C,1) method of Fourier series.

REFERENCES

[1] Alexits,G.,Convergence problems of orthogonal series,PergamonPress, London (1961).

[2] Lal,S.and Singh,H.P.,The degree of approximation of almost Lipschitz function by (N,p,q) (E,1) means,Int.Math. Forum, 5(2010), no.34, 1663-1671.

[3] Qureshi,K., On degree of approximation of a periodic function by almost Norlund Means, Tamkang J. Math., 12(1981) no.1-35.

[4] Qureshi,K and Neha,H.K.,A class of functions and their degree of approximation Ganita, 41 (1990) no.1,37.

[5] Sahney, B.N. and Goel, D.S., on the degree of approximation of continuous function, Rachi Univ. Math., J.4 (1973), 50-53.